

# Interacting Quintessence

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February 1, 2008

## Abstract

We demonstrate that a suitable coupling between a quintessence scalar field and a pressureless cold dark matter (CDM) fluid leads to a constant ratio of the energy densities of both components which is compatible with an accelerated expansion of the Universe.

PACS number: 98.80.Hw

Keywords: Cosmology, accelerated expansion, quintessence

## 1 Introduction

There is a growing consensus among astrophysicists that we live in an accelerating Universe. On the one hand, high-redshift type Ia supernovae (SNIa)

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are significantly fainter than expected in a decelerating model (such as the Einstein–De Sitter) [1]. Although the statistics is still low and extinction by interstellar dust may partly account for their low brightness and no conclusive model of evolution of SNIa and their progenitors is still available, the acceleration scenario is gaining further ground [2]. On the other hand, while measurements of the average mass density of the Universe systematically fall below the critical density, about 0.3 or 0.4 in critical units (see e.g. [3] and references therein), the position of the first acoustic peak in the temperature anisotropy power spectrum of the CMB strongly suggests that the total energy density is critical or near critical [4]. Combining both results one may rule out a flat matter–dominated universe (with  $\Omega_M = 1$  and  $\Omega_\Lambda = 0$ ) as well as an open universe with no cosmological constant ( $\Omega_M = 0.3$  and  $\Omega_\Lambda = 0$ ) at high statistical level [5]. More generally, one is led to conclude that very likely (i) about two third of the energy of the Universe is “dark” (i.e., non-luminous and not subject to direct detection via dynamical methods), and (ii) connected to this exotic and elusive energy must be a negative pressure, able to violate the strong energy condition.

The immediate candidate for such exotic energy, a small cosmological constant  $\Lambda$ , poses however an embarrassing question: Why the energy density in cold dark matter (which in the absence of interactions redshifts as  $a^{-3}$ , where  $a(t)$  is the scale factor of the homogeneous and isotropic metric) and the constant energy associated to  $\Lambda$  are of the same order precisely today? For this to occur one must have fine–tuned initial conditions right after the inflationary epoch. This constitutes the so–called “coincidence problem” [6]. To overcome this hurdle it was suggested that a nearly homogeneous but time depending scalar field with negative pressure should replace  $\Lambda$ . This peculiar field, widely known as “quintessence”, was independently introduced by Ratra and Peebles [7] and Wetterich [8] well before the supernovae results were even suspected. Today a host of quintessence models are known both in the realm of general relativity (see e.g., [9], [10]) and in scalar–tensor theories [11].

The target of this letter is to clarify a specific aspect of the coincidence problem, namely to present an attractor type solution of the two-component dynamics which is characterized by a constant ratio of order unity of the energy densities of the CDM and quintessence components and at the same time admits an accelerated expansion of the Universe. The basic ingredient of the corresponding model is to assume a coupling between CDM and the quintessence scalar field. It is this assumption of an interacting quintessence component by which our analysis differs from most investigations in this field

which assume an independent evolution of CDM and the scalar field. “Coupled quintessence” models have been shown to be useful in handling the coincidence problem by Amendola et al. [10]. While the models of these authors assumed a specific coupling from the outset, our strategy here is different. We do not specify the coupling from the beginning. We *determine* its structure from the requirement that it shall admit a solution for the dynamics of the two-component system of CDM and quintessence with a constant ratio for the energy densities. This strategy seems legitimate since there does not exist any microphysical hint on the possible nature of a coupling between CDM and quintessence. It will provide us with a transparent phenomenological picture of the “final state” of the cosmic dynamics (for a less bleak eschatological scenario see [12]), leaving open, of course, the question of how this state is approached and whether or not our current Universe has already reached it.

## 2 Scalar field plus cold dark matter

Let us consider a two-component system with an energy momentum tensor

$$T_{ik} = \rho u_i u_k + p h_{ik} , \quad (1)$$

where  $h_{ik} = g_{ik} + u_i u_k$  and

$$\rho = \rho_S + \rho_M , \quad p = p_S + p_M . \quad (2)$$

The subscript S refers to the scalar field component, the subscript M to the matter component (i.e. CDM). The energy density and pressure of the scalar field are

$$\rho_S = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad \text{and} \quad p_S = \frac{1}{2} \dot{\phi}^2 - V(\phi) , \quad (3)$$

respectively. The splitting (2) implies that there is only one 4-velocity,

$$u^i = u_M^i = u_S^i = - \frac{g^{ij} \phi_{,j}}{\sqrt{-g^{ab} \phi_{,a} \phi_{,b}}} .$$

( $\phi_{,a}$  is assumed to be timelike.) We postulate that the components do not evolve independently but that there exists some interaction between them, described by a source (loss) term  $\delta$  in the energy balances

$$\dot{\rho}_M + 3H (\rho_M + p_M) = \delta , \quad (4)$$

and

$$\dot{\rho}_S + 3H(\rho_S + p_S) = -\delta . \quad (5)$$

The last equation is equivalent to

$$\dot{\phi} \left[ \ddot{\phi} + 3H\dot{\phi} + V' \right] = -\delta . \quad (6)$$

As already mentioned, we will not specify the interaction from the outset but constrain  $\delta$  by demanding that the solution to (4) and (5) be compatible with a constant ratio between the energy densities  $\rho_M$  and  $\rho_S$ . It is convenient to introduce the quantities  $\Pi_M$  and  $\Pi_S$  by

$$\delta \equiv -3H\Pi_M \equiv 3H\Pi_S , \quad (7)$$

with the help of which we can write

$$\dot{\rho}_M + 3H(\rho_M + p_M + \Pi_M) = 0, \quad (8)$$

and

$$\dot{\rho}_S + 3H(\rho_S + p_S + \Pi_S) = 0 . \quad (9)$$

The rewriting of Eqs. (4) and (5) into Eqs. (8) and (9), respectively, makes the dynamic equations formally look as those for two dissipative fluids. The fact that there is a coupling between them has been mapped onto the relation  $\Pi_M = -\Pi_S$  between the effective pressures  $\Pi_M$  and  $\Pi_S$ . Some early models of power law inflation also share this feature (see e.g., [13]). Below we shall map the interaction term  $\delta$  onto a corresponding interaction potential.

### 3 Attractor solution and cosmological dynamics

Consider now the time evolution of the ratio  $\rho_M/\rho_S$ ,

$$\left( \frac{\rho_M}{\rho_S} \right)' = \frac{\rho_M}{\rho_S} \left[ \frac{\dot{\rho}_M}{\rho_M} - \frac{\dot{\rho}_S}{\rho_S} \right] . \quad (10)$$

By introducing the shorthands

$$\gamma_M \equiv \frac{\rho_M + p_M}{\rho_M} = 1 + \frac{p_M}{\rho_M}, \quad \text{and} \quad \gamma_S \equiv \frac{\rho_S + p_S}{\rho_S} = \frac{\dot{\phi}^2}{\rho_S}, \quad (11)$$

we obtain

$$\left( \frac{\rho_M}{\rho_S} \right)' = -3H \frac{\rho_M}{\rho_S} \left[ \gamma_M - \gamma_S + \frac{\rho}{\rho_M \rho_S} \Pi_M \right] . \quad (12)$$

Obviously, there exists a stationary solution  $(\rho_M/\rho_S)^\cdot = 0$  for

$$\Pi_M = -\Pi_S = \frac{\rho_M \rho_S}{\rho_M + \rho_S} (\gamma_S - \gamma_M) . \quad (13)$$

Since the CDM behaves as dust, i.e.  $p_M \ll \rho_M$ , we find

$$\Pi_M \approx - \left[ 1 - \frac{\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V} \right] \frac{\rho_S \rho_M}{\rho} , \quad (14)$$

or, by virtue of  $\frac{1}{2}\dot{\phi}^2 - V = p_S \approx p$ ,

$$\Pi_M \approx \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \frac{\rho_S}{\rho} \rho_M = \frac{p}{\rho} \rho_M . \quad (15)$$

The coupling term corresponding to this is

$$\delta = 3H \left[ 1 - \frac{\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V} \right] \frac{\rho_S \rho_M}{\rho} , \quad (16)$$

or, equivalently,

$$\delta = -3H \frac{p_S}{\rho} \rho_M = -3H (\gamma_S - 1) \frac{\rho_S \rho_M}{\rho_S + \rho_M} . \quad (17)$$

Introducing the notation  $r \equiv \rho_M/\rho_S = \text{const}$  we may further write

$$\delta = -3H (\gamma_S - 1) \frac{\rho_M}{r+1} , \quad \text{or} \quad \delta = -3H (\gamma_S - 1) \frac{r}{r+1} \rho_S . \quad (18)$$

Invoking the Friedmann equation valid for universes with spatially flat sections,

$$3H^2 = 8\pi G [\rho_S + \rho_M] , \quad (19)$$

we have  $3H = \sqrt{24\pi G \rho}$ , and, consequently,

$$\delta = -\sqrt{24\pi G} (\gamma_S - 1) \frac{\rho_S \rho_M}{\sqrt{\rho_S + \rho_M}} . \quad (20)$$

With (18), in a spatially flat universe equivalent to (20), we have identified the interaction between the pressureless fluid (CDM) and the scalar field (quintessence) that guarantees a constant ratio  $r$  of the energy densities of both components.

To study the stability of this stationary solution against small perturbations we introduce the ansatz

$$\frac{\rho_M}{\rho_S} = \left( \frac{\rho_M}{\rho_S} \right)_{st} + \epsilon$$

into (12) -the subscript st is for “stationary”. The result is

$$\begin{aligned} \dot{\epsilon} &= 3H \left[ \left( \frac{\rho_M}{\rho_S} \right)_{st} + \epsilon \right] \left[ \frac{p_S}{\rho_S} - \frac{\rho}{\rho_S} \frac{\Pi_M}{\rho_M} \right] \\ &= 3H \left[ \left( \frac{\rho_M}{\rho_S} \right)_{st} + \epsilon \right] \left[ \frac{p_S}{\rho_S} - \left( 1 + \left( \frac{\rho_M}{\rho_S} \right)_{st} + \epsilon \right) \frac{\Pi_M}{\rho_M} \right]. \end{aligned} \quad (21)$$

The behavior of the perturbed solution depends on the ratio  $\Pi_M/\rho_M$ . For the stationary solution itself we may read off  $\Pi_M$  from (7) and (18). However, for deviations from stationarity an additional assumption is necessary. At first sight the most obvious choice seems to be  $|\Pi_M| \propto \rho_M$  also in the vicinity of the stationary solution. As to be seen from (7), the coupling term becomes asymmetric with respect to  $\rho_M$  and  $\rho_S$  under such conditions. It will turn out that a more appropriate choice is the assumption  $\Pi_M = -c\rho$ , where  $c$  is a constant  $c > 0$ . This type of interaction is symmetric in  $\rho_M$  and  $\rho_S$ . Up to first order in  $\epsilon$  we find in such a case,

$$\dot{\epsilon} = 3Hc \frac{r^2 - 1}{r} \epsilon. \quad (22)$$

This implies that the stationary solution is stable for  $r < 1$ , which is clearly compatible with the presently favored observational data  $\rho_M \approx 0.3$  and  $\rho_S \approx 0.7$ . Consistency with  $\Pi_M$  from (7) and (18) fixes  $c$ :

$$c = r \frac{1 - \gamma_S}{(1 + r)^2}. \quad (23)$$

The positivity of  $c$  is guaranteed for  $\gamma_S < 1$ .

With  $p \approx p_S$  today, the stability condition corresponds to [cf. Eq. (21)]

$$\frac{p}{\rho} - \frac{\Pi_M}{\rho_M} \leq 0. \quad (24)$$

Since we seek accelerated expansion, the total pressure  $p \approx p_S$  must be negative, i.e., the potential term must dominate the kinetic term, equivalent to  $\gamma_S < 1$ . It is remarkable that according to (23) this coincides with the condition for  $c$  to be positive. From (7) and (18) we find that a value  $\gamma_S < 1$

implies  $\Pi_M < 0$  and  $\delta > 0$ . There is a transfer of energy from the scalar field to the matter, which reminds of decaying vacuum energy approaches for the dynamics of the early universe (see, e.g., [20]). The stationary epoch  $\Pi_M/\rho_M = p/\rho$  has to be approached in such a way that

$$\frac{|\Pi_M|}{\rho_M} \leq \frac{|p|}{\rho} . \quad (25)$$

Since  $|\Pi_M|$  is proportional to  $\delta$ , this means, the interaction may be small as long as the system is still far from the attractor solution.

It is expedient to emphasize that the apparently subtle point to assume  $|\Pi_M| \propto \rho$  instead of  $|\Pi_M| \propto \rho_M$  is essential for the stability properties of the stationary solution. Namely, similar considerations as those leading to (22) show, that there does not exist a stable solution with accelerated expansion for  $|\Pi_M| \propto \rho_M$ . Therefore, a dependence  $|\Pi_M| \propto \rho$  is mandatory for a physically sensible solution. This represents a restriction on the type of interaction that produces a stationary ratio  $\rho_M/\rho_S$ . While for the stationary solution itself  $\Pi_M \propto \rho_M$  and  $\Pi_M \propto \rho$  are not really different since  $\rho_M \propto \rho$ , the difference becomes crucial if one perturbs the solution.

Note that the stability is connected to the presence of an effective dissipative stress in the matter fluid. This parallels the result that the scalar field needs the assistance of a dissipative fluid stress for the coincidence problem to find solution in spatially flat accelerating Friedmann–Robertson–Walker models [14].

Given the interaction term (18), we may find the dependence of  $\rho_M$  and  $\rho_S$  on the scale factor. Because of  $p_M \approx 0$ , Eq. (4) with (18) yields

$$\dot{\rho}_M + 3H\rho_M = -3H(\gamma_S - 1) \frac{\rho_M}{r+1} , \quad (26)$$

while (5) with (18) results in

$$\dot{\rho}_S + 3H\gamma_S\rho_S = 3H(\gamma_S - 1) \frac{r}{r+1} \rho_S . \quad (27)$$

Assuming  $\gamma_S$ , which is in the range  $0 \leq \gamma_S \leq 2$ , to be (at least piecewise) constant, we obtain

$$\rho_S \propto a^{-\nu} , \quad \rho_M \propto a^{-\nu} , \quad \nu = 3 \frac{\gamma_S + r}{r+1} . \quad (28)$$

Both energy densities happen to redshift at the same rate because we have chosen  $\delta$  to correspond to the stationary state. With the relationship  $\rho \propto$

$a^{-\nu}$  we can solve the Friedmann equation (19) to find

$$a \propto t^{2/\nu} \quad \Rightarrow \quad q \equiv -\frac{\ddot{a}}{aH^2} = -\left(1 - \frac{\nu}{2}\right). \quad (29)$$

The total energy density redshifts as  $\rho \propto t^{-2}$ , independently of  $\gamma_S$  and  $r$ . Power law accelerated expansion will occur for  $\nu < 2$ , equivalent to

$$r + 3\gamma_S < 2. \quad (30)$$

Together with the above derived stability condition  $r < 1$  this amounts to  $\gamma_S < 1/3$  or  $p_S/\rho_S < -2/3$  for accelerated expansion.

Defining

$$\Omega_M \equiv \frac{8\pi G}{3H^2}\rho_M, \quad \text{and} \quad \Omega_S \equiv \frac{8\pi G}{3H^2}\rho_S, \quad (31)$$

we have

$$\Omega_M = \frac{r}{r+1}, \quad \text{and} \quad \Omega_S = \frac{1}{r+1}, \quad (32)$$

respectively, and also

$$\Omega_S = \frac{8\pi G}{3} \frac{\nu^2}{4} \rho_S t^2. \quad (33)$$

For  $\rho_S$  we find

$$\rho_S = \frac{1}{6\pi G} \frac{1+r}{(\gamma_S+r)^2} \frac{1}{t^2}. \quad (34)$$

Combination with (11) yields

$$\dot{\phi} = \sqrt{\frac{\gamma_S(1+r)}{6\pi G}} \frac{1}{(\gamma_S+r)} \frac{1}{t}, \quad (35)$$

i.e.,  $\phi$  evolves logarithmically with time. Furthermore, with the help of (3) and (11) it follows that

$$\rho_S = \frac{2V(\phi)}{2-\gamma_S} = \frac{\dot{\phi}^2}{\gamma_S}, \quad (36)$$

which together with (34) and (35) leads to

$$V(\phi) = \frac{1}{6\pi G} \left(1 - \frac{\gamma_S}{2}\right) \frac{1+r}{(\gamma_S+r)^2} \frac{1}{t^2}. \quad (37)$$

Since

$$V'(\phi)\dot{\phi} = \dot{V}(\phi) = -2\frac{V}{t}, \quad (38)$$



by virtue of (35) we obtain

$$V'(\phi) = -\lambda V(\phi) , \quad (39)$$

where

$$\lambda = \sqrt{\frac{24\pi G}{\gamma_S (1+r)}} (\gamma_S + r) \quad (40)$$

and, consequently,

$$V(\phi) = V_0 \exp [-\lambda (\phi - \phi_0)] . \quad (41)$$

By similar steps one shows that the interaction term  $\delta$  in Eq. (6), given by the second expression in (18), may be mapped onto an interaction potential  $V_{int}$ :

$$\frac{\delta}{\dot{\phi}} \equiv V'_{int} \quad \Rightarrow \quad V_{int} = -\frac{2r}{\gamma_S + r} \frac{1 - \gamma_S}{2 - \gamma_S} V(\phi) . \quad (42)$$

Introducing an effective potential

$$V_{eff} \equiv V(\phi) + V_{int} , \quad (43)$$

the equation of motion for the  $\phi$  field becomes

$$\ddot{\phi} + 3H\dot{\phi} + V'_{eff} = 0 . \quad (44)$$

It is rather reassuring (although not unexpected, cf.([15]) to find a potential (43) with (41) and (42), substantially backed by some field theories. It appears for instance in  $N = 2$  supergravity [16]. Likewise, linear combinations of exponential potentials naturally arise in theories undergoing dimensional compactification to an effective 4-dimensional theory; it is reasonable to expect that one of them will eventually dominate [17].

With the help of (30) the condition for accelerated expansion becomes

$$\lambda^2 < 24\pi G \frac{(1 - \gamma_S)^2}{(1 + r) \gamma_S} . \quad (45)$$

This is similar but not identical to conditions which have been obtained for corresponding solutions in the non-interacting case [8, 18, 19] or for different types of coupling [10, 13, 18, 21]. These authors started with an exponential potential in which  $\lambda$  is a free parameter initially. Then they investigated the parameter range for which there exists an attractor solution which is also inflationary. Our strategy is different insofar, as we have first constructed

a solution with the required properties and then read off the corresponding parameter combination.

Notice also that the way the attractor is approached remains open (only that in order to guarantee stability the approach, according to (25), has to proceed from a smaller coupling than given by the stationary solution itself).

## 4 Discussion

We proposed a coupling  $\delta$  (given by (17), (18), or (42) with (41)) between a quintessence scalar field and a CDM fluid that leads to a stable, constant ratio for the energy densities of both components, compatible with a power law accelerated cosmic expansion. This interacting quintessence approach indicates a phenomenological solution of the coincidence problem that afflicts many attempts to cope with late acceleration (especially those based in a cosmological constant). Unlike other approaches the potential is not an input but derived from the coupling. It remains to be seen to what extent this potential is consistent with measurements of the supernovae distances [22] once the SNAP satellite comes up with enough SNIa statistics [23]. Alternative and possibly earlier available tests rely on the Alcock-Paczyński test for quasar pairs [cf. Ref. [24]].

While focusing on the stationary solution straightforwardly provides us with an expression for the interaction which realizes a corresponding state, we mention again that this procedure leaves open how this interaction is exactly “switched on” in order to account for the necessary transition from the era of decelerated expansion to that of accelerated expansion. The coupling should be ineffective until the condensation of protogalaxies has entered the non-linear regime. In a sense, this feature reminds of the “exit problem” of many inflationary models. There are attempts to tackle this problem with the help of a specific coupling function between  $\phi$  and CDM together with a separately postulated exponential potential [10]. However, a really satisfactory solution is still missing. What one would like to have is an interaction which is negligible in the matter dominated era and asymptotically approaches (17) for large times. We hope that our stationary solution will give an indication for a quintessence–CDM coupling that, aside from characterizing the stationary state of the late accelerated expansion, smoothly joins the previous matter–dominated era of decelerated expansion when one goes backward in time.

## Acknowledgments

This work was supported by the Deutsche Forschungsgemeinschaft, the Spanish Ministry of Science and Technology under grant BFM 2000-0351-C03-01, the University of Buenos Aires under project X223-2001-2002, and NATO.

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